Large-Scale Distribution Planning—Part I: Simultaneous Network and Transformer Optimization

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***Abstract*—This paper is the first of two and presents a planning methodology for low-voltage distribution networks. Combined optimization of transformers and associated networks is performed, considering the street layout which connects the different consumers. In this first part, micro-optimization, the planning zone is divided into small zones, mini-zones, which are optimized independently. A repetitive procedure is used in order to locate transformers using clustering techniques. Optimum capacity, customers to be satisfied, the optimum network to be used and losses associated to this network are determined for each location. The methodology is applied over an area of 12.9 km^2 with nearly 20 215 consumers.**

In the second paper, two macro-optimization methodologies are discussed based on the planning results for each mini-zone, one based on the Voronoi polygons in order to improve load grouping into mini-zones and the other based on the combination of neighboring networks into a single transformer by means of a Tabu search. Finally, the methodology is applied to a zone with a surface of 2118 km^2 and approximately 1 300 000 customers.

***Index Terms*—Cluster, low voltage, network planning, power distribution planning.**

I. Introduction

DISTRIBUTION system planning seeks to determine the set of optimum installations for supplying a set of loads spatially distributed over a geographic zone. Specifically, the size and location of substations, the layout of the associated network and the type of conductors must be determined. Available transformers and conductors must be used, recognizing their capacity limits and network voltage drop restrictions.

If planning focuses on medium voltage, the substations must be high-voltage/medium-voltage and the networks shall correspond to feeders which supply the distribution transformers. If planning focuses on low voltage, the substations shall correspond to distribution transformers and the networks which make up the necessary layout to supply final customers. These problems have been approached by means of mathematical programming and heuristic algorithm techniques.

When using mathematical programming there are procedures which resolve substation planning [1]–[4], network optimization [5]–[8], and the joint planning of substations and network. Branch and bound is used in [9], mixed whole programming is used in [10] and [11]. Continuous variables are used in [12] and [13] to enable application of the Lagrangian method. In [14]–[16] the characteristic dynamics of the problem are approached through linear programming, Benders decomposition and mixed integer programming, respectively. In [17] medium and low-voltage network combined planning is approached by means of mixed integer linear programming.

Notwithstanding, as the result of the combinatory nature of the problem, the aforementioned formulations are only able to solve small problems. For this reason, heuristic algorithms have been developed in order to approach problems of a larger scale. Therefore, in [18]–[22], the branch exchange technique is used in order to determine the optimum network. In [23]–[25], Tabu search is used to solve the problem and in [26] it is solved by means of ant colony optimization. Genetic algorithms are used in [27]–[34] in order to find good solutions for dynamic problem considerations and/or multiobjective formulations. The planning of a low voltage network is done in [32], considering the relation with the mid voltage network, and the link between transformer and network; however, it corresponds to a rural network with few nodes and it does not consider the street layout.

The present study seeks to consider, through a heuristic optimization algorithm, the existing interaction between distribution transformers and the supply low-voltage network, recognizing the layout possibilities between the different loads. This is applied over a large-scale real network in a Greenfield planning exercise, where only the location and load size of consumers is known throughout the study timeframe.

The problem is specifically approached in this first part, dividing the planning zone into smaller zones, hereinafter mini-zones, performing an optimization process in each of them, known as micro-optimization. A macro-optimization algorithm is applied in the second part, enabling an overall optimization of the planning zone.

II. First Stage

The street layout must first be considered in order to reach a feasible solution, since this will prevent conductors from being routed through restricted areas, under the assumption that if a street can be routed then a conductor can also be routed. In addition, street groups with no road network to connect them must be identified. For this purpose, all connected components in graph $G(V,E)$ are determined, where V represents the vertices of street segments and E represents the street segments. The graph is connected if any two vertices belonging to V are connected by a k longitude street with $k:\{1,\ldots,E\}$, k is called the longitude path between the vertices v_{o} and v_k belonging to V, and a series of vertices and edges in the form $\{(v_{\mathrm{o}},v_1),(v_1,v_2),(v_i,v_{i+1}),\ldots,(v_{k-1},v_k)\}$, from the vertex

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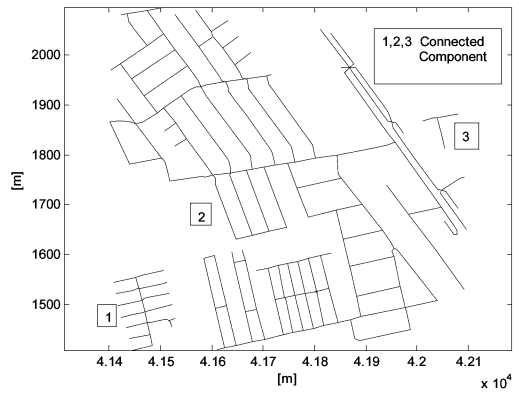

Fig. 1. Connected subgraphs into mini-zone.

v_o to the vertex v_k which has k edges which are different from each other.

In order to determine the connected components, which in this work are called connected subgraphs, a vertex is taken from the mini-zone and all possible streets coming out of the same are analyzed. All vertices and arcs thus reached belong to the connected subgraph. If any vertex of the mini-zone were to be omitted, this means that there is more than one subgraph. In order to determine it, the same procedure is followed, but this time using a vertex that was not included in the former subgraph as the initial vertex. The procedure is repeated until all vertices in the mini-zone belong to some connected subgraph (Fig. 1).

III. Micro-Optimization

The planning zone is divided into regular mini-zones of side length n. Connected subgraphs which make up each mini-zone are determined. Each subgraph requires loads to be grouped into one or more subsets, placing and assigning a transformer to each. Clustering is used in order to perform this grouping in an intelligent manner, avoiding complete enumeration. For example, in [33] a cluster algorithm is applied in order to determine the location and size of feasible substations.

This study has chosen to use a clustering technique known as k-means, which enables the classification of a set of m loads in k subsets. The simplified algorithm is:

1) choice of k loads from the m to be grouped; these constitute the initial k centroids. Each load is assigned to the closest centroid according to Euclidian distance, creating k groups;
2) a new centroid is calculated for all loads assigned to each of the groups;
3) Steps 2 and 3 are repeated until an established convergence criterion has been satisfied.

Convergence criterion is given by the minimum between maximum number of allowed iterations and the distortion variation considering two successive iterations. Distortion corresponds to the sum of all distances between loads and their respective centroid.

This methodology requires knowledge of the number of transformers to be installed. In [35], k-means is used as the final stage, in order to place a number of previously-selected transformers by means of mathematical programming, but it does not consider distributed demand. Thus, transformers which represent optimum performance for a specific load may cease to do so when street connection restrictions and expenses associated to each network are included.

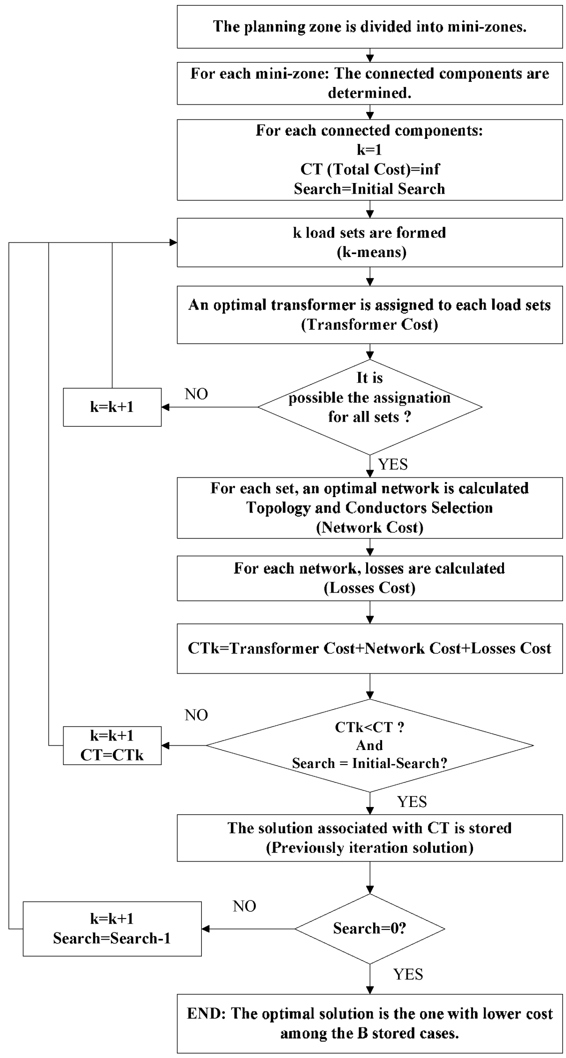

Fig. 2. Micro-optimization algorithm.

In order to recognize the existing relationship between transformer capacity and location with the cost of its associated network, the proposed methodology (Fig. 2) is applied to each connected subgraph.

The proposed methodology begins with the placement of a transformer at the load center; then the cost of the associated network is calculated, considering street topology, and the cost of the transformer which meets this demand. Subsequently, the same procedure is used for the case of two transformers. The number of transformers is successively increased until the optimum number is found. This prevents an a priori choice of the number of transformers since each iteration provides the right number to be installed.

This methodology is based on the consideration that as the number of transformers increases, the costs associated to the network reduce and transformation costs increase, a situation which allows finding a minimum number.

Each of the micro-optimization process stages will be explained in detail as follows.

A. Transformer Placement and Assignment

Preliminary placement of transformers and their loads is performed using k-means, taking only load location and the number of transformers to be installed as input data. Once the preliminary locations have been identified, the transformers are relocated in the closest street segment, such that its connection to the associated load considers street network topology.

The following steps are followed in order to determine the capacity of each transformer.

1) The average demand to be satisfied by the transformer in the base year is calculated. The energies of each assigned load are added and then divided by the number of annual hours.
2) Load factor is calculated based on the number of customers associated to the transformer (Appendix A).
3) Maximum demand (D_{MAX}) corresponding to load for the base year is calculated based on the average demand (D_A) and the load factor (*lf*).

$$lf = \frac{D_A}{D_{MAX}} \Rightarrow D_{MAX} = \frac{D_A}{lf}. \tag{1}$$

4) The most economic transformer (Appendix B), able to supply the maximum demand evolution throughout the study timeframe, is assigned.

If one of the transformers covers a demand higher than its capacity, the k-means process shall be repeated but this time with one additional transformer. This process shall be repeated until all groups have a transformer assigned to the same.

B. Network Topology

The most commonly used method in order to determine the network layout is the construction of a minimum expansion tree with root in the transformer location. However, this technique does not provide a feasible solution since it does not consider restricted access areas, which are avoided if the street layout is considered. Therefore, load projections over streets are used, instead of their real position, in order to guide the algorithm's path. In addition, street segment vertices are included as auxiliary nodes in order to guarantee bifurcations only at segment intersections. The following algorithm is used.

1) The tree is initialized at the transformer location (tree root).
2) The real or auxiliary node which has not yet been included is added to the tree which is closest to any of the tree nodes, as long as that node belongs to the same segment.
3) Step 2 is repeated until all segment nodes have been included.
4) The node which has not yet been included, and is the closest to the vertices of the set of segments which have already been included and belong to a neighboring segment, is added to the tree. Two or more segments are neighbors if they share a common vertex.
5) Steps 2 and 3 are repeated for the new segment included.

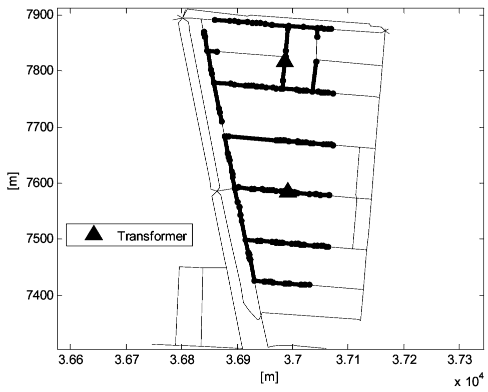

Fig. 3. Network topology.

6) Steps 4 and 5 are repeated until all load projections over street segments have been added.
7) Finally, the auxiliary nodes which do not connect real nodes are eliminated.

The preliminary location of the transformer corresponds to the load center of the supplied loads (Fig. 3). However, its projection over the street segment does not necessarily correspond to the network load center. Therefore, once an optimum layout has been determined, it is proposed that the transformer be relocated at the best balanced node, which is to say, where flows coming out of their branches are similar (Fig. 4). The simplified algorithm is as follows:

1) Flows coming out of both transformer branches are calculated. Balance is defined as the absolute difference between flows.
2) The transformer is placed at the offspring node with the highest flow.
3) The number of branches coming out of the new position is analyzed.

   If the number of branches is 2

   a) Flows coming out of both branches are calculated.
   b) Steps 2 and 3 are repeated until the balance is no longer reduced.

   If the number of branches is greater than 2

   c) Starting from the offspring of the new position, the process goes back to 3, as long as the balance is improved through the branch.
4) The process ends when it is no longer possible to follow any path to improve balance.

C. Optimum Conductor Selection

In [7] the conductor is optimally chosen by taking into consideration the network layout and resultant voltage drops. However, at low-voltage networks where the number of loads is high, such process increases execution time. This is so because of the need to consider at each iteration, the effect of the choice of a certain conductor for the rest of the network. This situation can be avoided by indirectly considering the effect of voltage drops through the cost of network losses, and only finding a cost curve for each conductor section, in terms of the current going through [8].

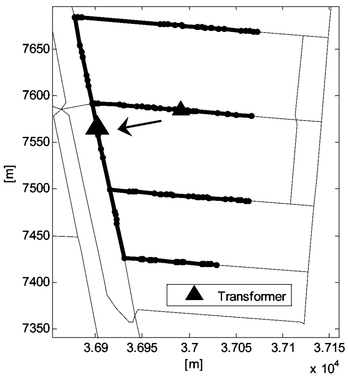

Fig. 4. Network balance.

The results obtained at [36] are used in this study, starting with the following expression (2):

$$NPV_i(I) = INV_{i,0} + \sum_{k=0}^{T} \frac{CL_{i,k} \cdot (1 - Timp) - Dep_{i,k} \cdot Timp}{(1+r)^k} \quad (2)$$

where

$NPV_i(I)$	net present value of the type i conductor for a current I;
$INV_{i,0}$	initial investment in a type i conductor;
$CL(I)_{i,k}$	cost of i conductor losses in year k for an I current value;
Timp	tax rate (17%);
$Dep_{i,k}$	i conductor depreciation in year k;
r	discount rate (10%);
T	number of years in the study timeframe (15 years).

Therefore, if a certain current I, circulates in one segment of the network, assessment of I in (2) is enough for all sections of all conductors whose thermal limits support I. In this manner, the optimum conductor is that which costs less among the candidate sections.

When it comes to Greenfield planning, it is impossible to determine the current of each segment, since the conductors have not been identified. For this reason, it is determined by means of a simplified process which takes advantage of the radial nature of the problem and assumes a balanced system [21], which enables single phase analysis. For each segment of the network, each phase complies with the following:

$$P_i = \frac{r_i \cdot \left(P_i^2 + Q_i^2\right)}{\left|V_{U(i)}\right|^2} + PL_i + \sum_{k=1}^{R} P_k \quad (3)$$

$$Q_i = \frac{x_i \cdot \left(P_i^2 + Q_i^2\right)}{\left|V_{U(i)}\right|^2} + QL_i + \sum_{k=1}^{R} Q_k \quad (4)$$

where

P_i	active power at the beginning of segment i;
Q_i	reactive power at the beginning of segment i;
r_i	resistance of segment i;
x_i	reactance of segment i;
$V_{U(i)}$	upstream voltage of node i;
PL_i	active demand of node i;
QL_i	reactive demand of node i;
R	branches which come out of node i.

Since this is a distribution system, losses are much less than the flows which circulate through the lines [21] and therefore the quadratic terms of (3) and (4) can be neglected in the following:

$$P_i = PL_i + \sum_{k=1}^{R} P_k \quad (5)$$

$$Q_i = QL_i + \sum_{k=1}^{R} Q_k. \quad (6)$$

Independence can be observed at (5) and (6) both in terms of voltage and the type of conductor used. This enables quick and simple calculation, starting from the extreme nodes and sequentially adding upstream loads. Once flows have been identified, an approximate current is calculated for each segment, as the quotient between apparent power magnitude and nominal phase-neutral voltage.

D. Loss Calculation

The cost of energy losses is incorporated into the objective function, because losses implicitly include the voltage drop into the optimization process, since they act as a voltage drop penalization [15].

Upon identifying the conductor type and length of each network segment, losses can be determined regarding the coincident demand for each transformer. Subsequently, in order to estimate annual energy losses (8), the loss load factor, *lsf*, is used [37], which depends on the load factor, *lf*, by means of the following:

$$lsf = 0.08 \cdot lf + 0.92 \cdot lf^2 \quad (7)$$

$$L_{ENERGY_i} = 8760 \cdot lsf \cdot L_{POWER_i} \quad (8)$$

where

L_{ENERGYi}	annual energy losses in network i for the base year;
L_{POWERi}	power losses in the maximum demand scenario in network i for the base year.

Energy losses represent energy which is not sold throughout the entire study timeframe and therefore must be assessed at the distributor purchase price. In addition, it must be considered that demand grows, which implies increasing losses. Therefore, the cost of losses is given by the following:

$$C_{LOSSES} = C_{ENERGY} \cdot \sum_{i=1}^{N} \sum_{j=1}^{T} \frac{L_{ENERGY_i} \cdot (1+g)^j}{(1+r)^j} \quad (9)$$

where

C_{LOSSES}	total cost of network losses for the timeframe;
C_{ENERGY}	energy unit cost;
N	number of networks;
T	number of years in the study timeframe;
g	annual growth rate of losses;
r	annual discount rate.

E. Search

Using the aforementioned procedures, the overall planning cost can be calculated for a complete iteration of the proposed algorithm. Such cost includes transformer and conductor investment costs plus the cost of network-associated losses.

If N transformers are installed during the first iteration for a total cost of C_{N}, the following iteration will follow the same procedure with $N+1$ transformers for a new overall cost of $C_{\mathrm{N+1}}$. This cycle is repeated as many times as necessary until the search has been completed. The search is defined as the number of additional iterations performed after finding the first minimum cost related to overall costs.

The first minimum cost is given by the transformers and networks configurations that gives overall costs lower than in the next iteration. The number chosen as search is indicated and justified in the Section IV.

Finally, the plan chosen as optimum for the connected subgraph is that which represents the lowest overall cost among all iterations.

IV. Micro-Optimization Results

The procedure described is applied over a planning area corresponding to the district of Macul, located in Santiago, Chile. The district is made up of 20 215 consumers distributed over a surface of 12.9 km^2 which is divided into regular mini-zones measuring 500 m by 500 m (Fig. 5). The planning horizon is 15 years. The following results were obtained from the application.

A. Network Balance

Three cases are analyzed in order to determine whether the incorporation of network balance leads to improved optimization. These cases are assessed for the planning zone, considering 20 algorithm executions for each case (Fig. 6).

1) Case 1 does not relocate transformers. An average cost of CLP 1 906 197 901 (the currency used for this paper is the Chilean peso) is obtained in an average time of 3.05 min (in a PC Intel Pentium 4, 3.00 GHz).

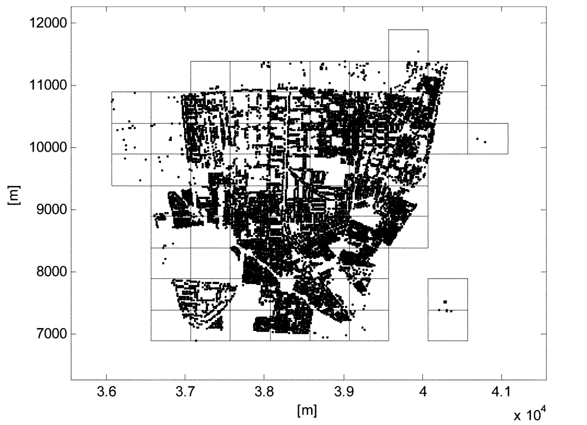

Fig. 5. Planning zone.

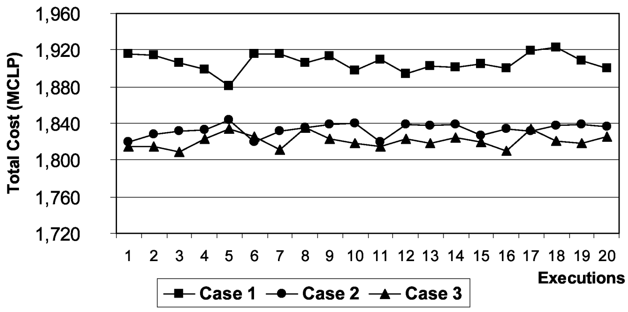

Fig. 6. Network balance effect.

2) Case 2, transformers are relocated after the best network topology has been chosen. An average reduction of 3.8% is obtained compared to case 1, in an average time of 3.56 min.
3) Case 3, balance is performed with each algorithm iteration. Average savings of 4.4% are achieved in an average time of 6.55 min.

Balance includes a reduction of overall costs and therefore should be used in the optimization process.

Case 3 is 0.7% more economic than case 2, but requires an 84% greater execution time. Therefore, it is decided that balance shall only be included after calculating the final topology, case 2, in order to apply the procedure to a larger zone.

B. Search Number

This paper assumes that the first minimum cost corresponds to the overall minimum cost of the objective function. This is based on the fact that if a transformer is added to a zone, this will increase the transformation cost due to two effects:

1) The installed capacity increases (Appendix C).
2) The cost per capacity unit increases, due to the fact that units are less than or equal to those existing prior to the additional transformer. Therefore, as the result of economies of scale, each kVA installed is more expensive than the kVA existing in the previous iteration.

This cost increase is combined with a network cost reduction, due to reduced section size and therefore reduced costs for the conductor used. Notwithstanding, in the iteration that

TABLE I
SEARCH V/S TOTAL COST

Search	Total Cost (%)	Time (minutes)
0	100.0	3.6
1	99.8	3.9
2	99.9	4.3
3	101.1	5.0
4	98.3	5.3
5	100.3	5.7
6	100.4	6.7
7	99.6	7.2
8	100.7	7.6
9	99.6	8.7
10	99.5	9.3
11	100.4	10.3
12	99.6	11.5
13	100.4	12.6
14	100.3	13.7

TABLE II
ALGORITHM EXECUTIONS

Execution Number	Total Cost (%)	Time (minutes)	Execution Number	Total Cost (%)	Time (minutes)
1	99.8	3.6	**11**	99.3	3.4
2	99.7	3.6	**12**	100.3	3.6
3	99.9	3.5	**13**	100.2	3.5
4	100.0	3.5	**14**	100.3	3.4
5	100.5	3.6	**15**	99.6	3.4
6	99.3	3.6	**16**	100.0	3.5
7	99.9	3.5	**17**	99.9	3.5
8	100.1	3.6	**18**	100.2	3.7
9	100.3	3.4	**19**	100.3	3.5
10	100.4	3.5	**20**	100.2	3.6

increases overall costs it is assumed that conductor savings do not cover the transformation cost. This cost will probably not be covered in the following iterations, because every additional transformer increases the cost more than proportionality due to the economies of scale. It can be ratified by applying the methodology proposed to the analysis zone, considering different search values (Table I). 100% corresponds to a total cost of CLP 1 836 355 360.

It has been observed that search increases lead to greater execution times, without implying reductions in overall costs. Therefore the use of additional iterations is not justified after finding the first minimum cost.

The small existing differences can be attributed to the nondeterministic feature of the proposed methodology.

C. Convergence

The methodology proposed is nondeterministic, which is largely due to the clustering technique used for transformer location. Table II indicates the results of 20 methodology executions, in which 100% represents the average overall cost of each launch, which amounted to CLP 1 833 157 697, with an average execution time of 3.52 min.

Table II indicates the similarity of results found. In effect, the standard deviation of costs amounts to 0.34%, which enables decision-making based on the proposed model.

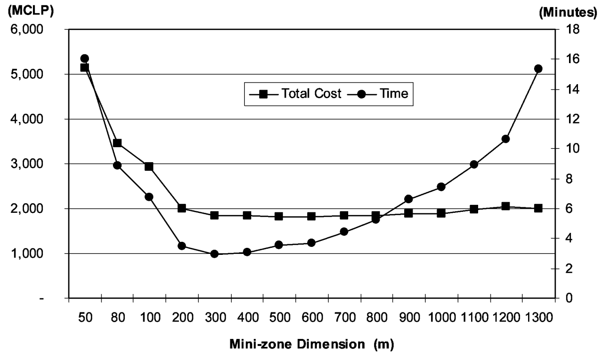

Fig. 7. Mini-zone size.

D. Size of the Mini-Zone

The right size of each mini-zone is determined using the micro-optimization process. The planning zone is divided into regular mini-zones of side length n, using different n values (Fig. 7).

Execution time is reduced when the division is between 300 and 400 m and reduced costs are produced when the division is between 500 and 600 m. Therefore, the best balance between execution time and solution quality is with 500 m, since this enables handling a greater universe of feasible solutions, in a period slightly larger than that of the 400 m case.

V. CONCLUSIONS

A methodology for Greenfield planning in a real-scale zone of a distribution system was developed. The existing relationship between transformation capacity and network costs was considered and street restrictions were taken into account. The process was based on dividing the planning zone into smaller zones, resolving the problem of planning distribution networks for each. It was shown that better results, without unnecessary increases in execution time, are achieved when using a search from zero and considering the network balance process after tracing the optimum topology. In addition, validity of the proposed algorithm was confirmed, given the fact that although the procedure is not deterministic, dispersion between its different executions is small (Table II) and therefore allows finding a good solution to the problem.

Finally, division into mini-zones enables independent optimization of them and therefore they could be processed in parallel, which would significantly reduce solution time.

APPENDIX

A) Load Factor Assessment: In this paper load factor is estimated by means of the following curve [36]:

$$lf = \begin{cases} \alpha \cdot Customers^{\beta} - \delta, & \text{if Customers} \leq 500 \\ 0.4, & \text{in other case} \end{cases} \quad (10)$$

where

lf	load factor of a group of customers;
Customers	number of customers in the group;
α	0.1687;

β 0.1577;

δ 0.0633.

B) Determination of Transformer Cost: The transformer which supplies a given load shall be that which does so at a minimum cost, including both investment expenses and loss expenses. In this manner, total expense [34], CT_k, is

$$CT_k = CF_k + CV_k \cdot P_k^2 \tag{11}$$

where

CF_k investment, installation, and fixed loss expenses in the k transformer;

CV_k electricity loss expenses in the k transformer;

P_k power injected by the k transformer.

It has been considered that the k transformer can only supply a demand between 0 kVA and its nominal capacity.

The variable cost represents the present value of loss expenses produced during the study timeframe and it is different for each of the feasible k transformers. In this paper, the results of [36] are used, where an energy cost of 18.5 CLP/kWh was considered, with a 15-year timeframe and a 30-year service life.

C) Installed Capacity Increase: Consider a planning zone with a transformer which supplies a set of C customers with E energy consumption, a load factor $lf\,(C)$ and a power requirement P. If it is modified and it is supplied by two transformers which take C_1 and C_2 customers with E_1 and E_2 energy demands, respectively, the installed power increases.

Given the fact that C_1 and C_2 are smaller than C, it is considered from (10)

$$\begin{aligned} lf(C) > lf(C_1) &\Rightarrow \frac{1}{lf(C_1)} \\ &> \frac{1}{lf(C)} \Rightarrow \frac{\frac{E_1}{T}}{lf(C_1)} > \frac{\frac{E_1}{T}}{lf(C)} \end{aligned} \tag{12}$$

$$\begin{aligned} lf(C) > lf(C_2) &\Rightarrow \frac{1}{lf(C_2)} \\ &> \frac{1}{lf(C)} \Rightarrow \frac{\frac{E_2}{T}}{lf(C_2)} > \frac{\frac{E_2}{T}}{lf(C)}. \end{aligned} \tag{13}$$

Adding (12) and (13)

$$\frac{\frac{E_1}{T}}{lf(C_1)} + \frac{\frac{E_2}{T}}{lf(C_2)} > \frac{\frac{E}{T}}{lf(C)} \Leftrightarrow P_1 + P_2 > P \tag{14}$$

where P_i is the maximum power for the i transformer {1,2}.

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